

Contra Scotching Dutch Books - 27/Jun/2008

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This short paper faces just one of the many criticisms that can be raised against the Dutch Book Argument.¹

The ideal lesson of the Dutch Book Argument (DBA) would be that the rational agent's degrees of belief satisfy the probability axioms². This lesson should serve to justify the Bayesian posit according to which probability has to be identified with those degrees of belief.

The original formulation of the DBA is normally traced back to independent works of de Finetti and Ramsey and it is centered on the notion of fair price. The fair price is the price $q_j(\$)$ for which an agent would be indifferent between buying or selling the access to a bet where the payoff is 1\$ if \mathcal{H}_j is conclusively verified or nothing otherwise.³ The fair price is standarily assumed to be the measure of the degrees of belief; Howson and Urbach⁴ write: 'Characterising degrees of belief, as we have done, in terms of attitudes to actual or potential bets is now so common as to be traditional'.

Dutch book is the particular type of betting contract in which the agent is necessarily subject to a sure loss for whatever hypothesis might be verified. If the agent offers a set of fair prices that cannot be open to a Dutch book, then those fair prices are said to be coherent.

In the DBA the agent has to provide his fair prices for the hypotheses at stake, and a bookie decides whether the agent must buy or sell the access for compulsorily betting on each of those hypotheses with those fair prices. Under these conditions – and possibly also some implicit other ones – here is the undisputed formal part of the DBA: if, and only if,

¹For an extensive analysis see my manuscript 'Gambling is bad' downloadable from my web site www.luigigobbi.com.

²The standard probability axioms are:

I. (Non-negativity) $p(\mathcal{H}_i) \geq 0 \quad \forall \mathcal{H}_i \in \Sigma$

II. (Normalization) $p(\top) = 1$ whereas \top is a tautology in classical logic

III. (Additivity) $p(\mathcal{H}_1 \vee \mathcal{H}_2 \vee \dots \vee \mathcal{H}_i \vee \dots) = p(\mathcal{H}_1) + p(\mathcal{H}_2) + \dots + p(\mathcal{H}_i) + \dots$ whereas \mathcal{H}_i 's are mutually exclusive ($\forall i, l$ (with $i \neq l$): $\mathcal{H}_i \wedge \mathcal{H}_l \models \perp$).

Σ is a non-empty set of hypotheses closed under negation and disjunction.

³From this definition it rationally follows that the fair price is a number of the real interval $[0, 1]$ extremes included.

⁴(Howson & Urbach 1989, p. 58).

the agent respects the probability axioms, then there does not exist a set of fair prices, such that the bookie can choose whether the agent must buy or sell in such a way that the agent is Dutch-booked. This is a mathematical theorem, that I call “DBA theorem” and that sometimes goes under the name of Ramsey-deFinetti theorem⁵.

As playing for sure losses is irrational, the argument goes that: rationality leads to offer fair prices that are coherent, that is to say, that satisfy the axioms of probability. And as it was assumed that fair prices represent degrees of belief: the following is the conclusion of the DBA.

The rational agent’s degrees of belief, therefore, conform to the probability calculus.

A very simple case now suffices as an example of how it works and how it does not. Suppose to consider $\mathcal{H}_j = \top$ (so that, according to the probability axioms, $p(\mathcal{H}_j) = 1$), and the agent offers (what he believes to be) a fair price of 0.99\$ on \mathcal{H}_j for entering in the 1\$-payoff bet about \mathcal{H}_j . Now, another delicate Bayesian idealized condition – necessary to the DBA – is a plain logical omniscience mastered by the bookie, whilst the agent is humanly allowed to be non-omniscient and liable to make mistakes. For this reason, the fair price assessed by the agent could be better expressed as the “subjectively-fair”⁶ price, whereas the “objectively-fair” price could be the one assessed by the bookie.

The bookie would then decide that the agent must sell the access for betting on \mathcal{H}_j (for his 0.99\$), in order for the agent to return 1\$ as a payoff; accordingly, the agent would necessarily lose 0.01\$ from this gambling contract. The agent has been irrational to assess a fair price that makes his loss a necessity. If he chose the fair price of 1\$ then he wouldn’t have lost any money. The agent should have not offered a fair price of 0.99\$ because that didn’t ascertain the facts about \mathcal{H}_j , namely its being a tautology. Therefore, claims like⁷ ‘[...] according to the Bayesian account of rationality, what matters are not the facts, but rather what the agent takes the facts to be. More precisely, what matters in determining what the agent should do are the agent’s own probability assignments’ are improper. Moreover, the agent has to respect the probability axioms objectively: imagine that he believes the same \mathcal{H}_j to be a contradiction and therefore he offers a fair price of 0\$ in order

⁵(Gillies 2000, p. 59).

⁶(Howson & Urbach 1989, p. 56).

⁷(Hájek 2005, p. 11).

to comply with the probability axioms; the rational access to objectivity could appear to many as an insurmountable task, but I don't want to deal with this major epistemological problem here.

So here the DBA theorem has been exemplified: if, and only if, the agent respects the probability axioms objectively, then there does not exist a fair price⁸ such that he must face a necessary loss.

On the other hand, this conclusion of the DBA follows because it was held – implicitly, but arbitrarily – that the bookie is bad for always acting against the agent in fixing him to buy or to sell. In the case above, if the bookie was good and decided that the agent had to buy, then the agent would have had a guaranteed net gain of 0.01\$. And more in general there would be the “good-DBA theorem”: if, and only if, the agent respects the probability axioms objectively, then there does not exist a fair price such that he must face a necessary net gain.

Therefore, the usual DBA is invalid⁹ and stands on an arbitrary break of symmetry: opting for bad bookies instead of good bookies.

However, Alan Hájek believes¹⁰ that he managed to restore the force of the DBA simply by exchanging fair price with fair-or-favorable price¹¹.

In fact, bad and good bookies are asymmetric now. A necessary net gain would occur from paying the favorable price of 0.99\$ in order to access a bet that necessarily returns 1\$, but the good-DBA theorem, respectively modified, is not possible anymore because, even if the agent respects the probability axioms objectively, there does exist a fair-or-favorable price such that he must face a necessary net gain. Also, the DBA theorem still holds by using the same fair price as a fair-or-favorable price. But, in this way, no goodness of a bookie could interfere to change the usual conclusion of the DBA. So, the revised_H DBA¹² holds, while no revised_H good-DBA does.

Hájek mentions¹³ 32 “luminaries” of the philosophy of probability who improperly consider

⁸In this case, the hypothesis is only one and so is the number of members of the set of fair prices.

⁹(Hájek 2005, p. 2).

¹⁰(Hájek 2005, p. 17), (Hájek forthcoming, p. 5).

¹¹A price is favorable when lower than fair.

¹²H stands for Hájek.

¹³(Hájek forthcoming, p. 3).

just fair prices instead of fair-or-favorable prices.

Nonetheless, I sustain that Hájek's restoration has been achieved by another arbitrary break of symmetry: opting for fair-or-favorable prices instead of fair-or-unfavorable. The proof is trivial: a necessary loss would occur from paying the unfavorable price of 0.01\$ in order to access a bet that necessarily returns 0\$; even if the agent respects the probability axioms objectively, then there does exist a fair-or-unfavorable price such that he must face a necessary loss. Therefore, now the revised_G DBA does not hold, but only the revised_G good-DBA does: if and only if the agent respects the probability axioms, then there does not exist a fair-or-unfavorable price such that he must face a necessary loss. Hence, as no wickedness of a bookie could interfere, but only his goodness, it is rational to always not respect the probability axioms!

Therefore, the arbitrary idealization has just been moved from bad or good bookies to fair-or-favorable or fair-or-unfavorable prices, but still it has not been removed. I think that those luminaries were not so wrong on this point. I think that Hajek didn't remove the arbitrary break of symmetry in the suppositions: like other supporters of the DBA, he liked that conclusion of the DBA and he wanted to make it true by choosing the conditions convenient to him.

In fact I would like to argue about any strategy aiming at repairing the original formulation of the DBA in such a way that it becomes irreproachable¹⁴.

Many stipulations, more or less implicit, were needed for having a formulations of the DBA that returns the desired conclusion without trouble. Some of these are the agent's willingness or compulsion to enter in the bet, the idea that the agent's rationality could lead to an acknowledgment of objective tautologies¹⁵ and, subject of this paper, the wickedness of the bookie. Another fundamental implicit stipulation is that the rational agent's goal is to lose money as little as possible. It was implicit because – normally – in potential or actual bets, it is rational to aim at losing as little money as possible. However, if therefore that is the goal, then anything rationally justifies making money and the agent ought to

¹⁴Therefore the title of this paper could be 'Counter scotching Dutch books' instead of 'Contra scotching Dutch books?' (against Hájek's 'Scotching Dutch books?'), in the sense of reinforcing the Milne's thesis in his 'Scotching the Dutch book argument'.

¹⁵More in general it is implicitly supposed that the agent's rationality can bring to logical omniscience and to a probabilistic coherence over all the possible infinite sentences of a language.

be ready to do anything he can in order to not lose money. For this purpose, the agent ought to provide some prices with certain mathematical features. This is proved by the DBA theorem and the agent should be aware of this (in fact the agent should be rational to recognize it is a tautology). Thus, there could be plenty of cases – and there are in the literature¹⁶ – where the agent achieves his goal of carrying out a bet nicely, even though his betting quotients don't at all conform to his degrees of belief.

If the agent's first goal is to do a good bet, and not to do a good measure of his degrees of belief, then he is liable to not succeed in showing the actual properties of what he was to measure, namely the degrees of belief.

I try to simplify the DBA even further.

By knowing the DBA theorem and the bookie's wickedness, the agent knows that if he doesn't offer fair prices that respect the probability axioms (and disregarding whether the fair prices perfectly correspond to the quantification of his degrees of belief), then he loses money – which is a bad thing. The agent would rationally act in the very same way for another equivalent bad thing¹⁷: the bookie threatens the agent to kill him if, and only if, he offers fair prices that don't satisfy the probability axioms. Clearly it is supposed that the agent wants to live (equivalent to the ordinary case where he doesn't want to be Dutch-booked), and therefore rationality has led the agent to offer fair prices that respect the probability axioms (e.g., offering $q_j=1\$$ on $\mathcal{H}_j = \top$).

But but does this mean that rationality leads the agent's degrees of belief to respect the probability axioms?

Arguably not. In fact, the agent could have offered the same fair prices (e.g., again on $\mathcal{H}_j = \top$) for the following scenario: the agent is not killed¹⁸ if, and only if, he offers fair prices that do not satisfy the probability axioms and he acts irrationally (offering $q_j=1\$$). Here irrationality has led the agent to offer the same fair prices, which now would represent irrational degrees of belief (therefore, $1\$$ would represent the degree of belief on the single $\mathcal{H}_j = \top$ – degree of belief that would be both rational and irrational, impossibly).

Degrees of belief need not be identified with the fair prices of actual or possible bets,

¹⁶Over ten of them are surveyed in my 'Gambling is bad'.

¹⁷This sort of analysis can be found also in (Maher 1993, p. 101).

¹⁸Viz. Dutch-booked.

because the fair price the agent finds rational to offer may easily not conform to his degree of belief. No identity is easily endorsable. But if not, the DBA would need in its support a general theory that explains rationality, degrees of belief and objectivity, and link them together satisfactorily (and such a theory must still be done).

Despite the supposition of taking fair prices to reflect degrees of belief is as common as traditional (Howson & Urbach 1989, p. 58), I think it is too sensibly fallacious in general and crucial for preventing the construction of a flawless formulation of the DBA: so long as this supposition is used in any argument, that very argument can be shown to be criticizable.

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